Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Advanced Calculus

Subject Code :4SC03MTC1 **Branch: B.Sc.(Mathematics,Physics)**

Semester : 3 **Date :22/04/2016** **Time : 2:30 To 5:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $x^2 - y^2 = 4b^2xy$, find $\frac{dy}{dx}$. **(02)**

b) Verify Euler's theorem for $u(x,y) = x^3 + y^3 - 3ax^2y$. **(02)**

c) Prove that $\beta(m,n) = \beta(m,n+1) + \beta(m+1,n)$ **(02)**

d) Prove that $\Gamma(n+1) = n!$. **(02)**

e) Find asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$ parallel to coordinate axis. **(02)**

f) Prove that $y = e^x$ is everywhere concave upwards. **(02)**

g) State Euler's Theorem for homogeneous. **(01)**

h) Write the relation between Beta and Gama function. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Evaluate $\lim_{\substack{x \to 0 \\ y \to 0}} (x^3 + y^3)$. **(05)**

b) If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$ **(05)**

c) If $u = cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\ \frac{\partial u}{\partial x} + y\ \frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$ **(04)**

Q-3 **Attempt all questions** **(14)**

a) If z is a homogeneous function of x, y of degree n and $z = f(u)$, prove that $x\ \frac{\partial u}{\partial x} + y\ \frac{\partial u}{\partial y} = n\ \frac{f(u)}{f'(u)}$. **(05)**

b) If $z = u^2 + v^2$ and $u = at^2, v = 2at$ find $\frac{dz}{dt}$. **(05)**

c) If $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_1x_3}{x_2}, y_3 = \frac{x_2x_1}{x_3}$, find $\frac{\partial(y_1,y_2,y_3)}{\partial(x_1,x_2,x_3)}$. **(04)**

Q-4 **Attempt all questions** **(14)**

a) Verify $JJ' = 1$, if $x = e^v \sec u$ and $y = e^v \tan u$. **(05)**

b) If z is a function of x and y. If $x = e^u + e^{-v}, y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{x} - y\frac{\partial z}{\partial y}$. **(05)**

c) Find expansion of $\cos x \cos y$ in power of x, y up to fourth order terms. **(04)**

Q-5 **Attempt all questions** **(14)**

a) Discuss the maximum and minimum of $x^2 + y^2 + 6x + 12$. **(07)**

b) Using Lagrange method of undetermined multipliers find the point upon the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value. **(07)**

Q-6 **Attempt all questions** **(14)**

a) Evaluate: $\int_0^\infty \sqrt{x} e^{-\sqrt[3]{x}}\, dx$ **(05)**

b) Evaluate: $\int_0^1 x^4 (1 - \sqrt{x})^5\, dx$. **(05)**

c) Prove that $\beta(m, n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\, dy$. **(04)**

Q-7 **Attempt all questions** **(14)**

a) State and prove Taylor's series of two variable. **(07)**

b) State and prove Duplication formula. **(07)**

Q-8 **Attempt all questions** **(14)**

a) Find range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Find point of inflection in each case. **(05)**

b) Find asymptotes of the curve $y = \frac{x^2+2x-1}{x}$. **(05)**

c) Find asymptotes of the curve $x = \frac{t^2+1}{t^2-1}, y = \frac{t^2}{t-1}$. **(04)**